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FISICA PER TUTTI

Come insegnare la fisica degli tsunami
Teaching the physics of tsunamis

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Riassunto. Gli tsunami sono un importante fenomeno la cui conoscenza di base è essenziale per gli studenti di qualsiasi disciplina e anche per il grande pubblico. Questa conoscenza, infatti, può avere un impatto sulla consapevolezza generale dei rischi e soprattutto sull'adozione di efficaci contromisure preventive. Presentiamo qui l'aggiornamento di una strategia didattica cha ha già avuto degli effetti estesi e molto positivi nel corso di due decadi. Gli strumenti d'insegnamento sono discussi a differenti livelli, iniziando da argomenti molto semplici che individuano una serie di contromisure, e concludendo con due nuove derivazioni, formali ma elementari, della legge della velocità.

Abstract. Tsunamis are an important phenomenon, whose basic knowledge is essential for students with different backgrounds as well as for the general public. Such a knowledge, in fact, can impact the general awareness of the risks, and above all the adoption of effective preventive countermeasures. We present here an update of a didactic strategy that already had broad and very positive effects over two decades. The teaching instruments are discussed at different levels, starting from extremely simple arguments that identify a series of countermeasures, and concluding with two formal but elementary new derivations of the speed law.

1. A terrible tragedy revisited

On December 26, 2004, 7:58 a.m.: a 9.1-9.3 magnitude earthquake stroke for ten minutes the Indian Ocean not far from Sumatra in Indonesia [1–8]. It triggered a gigantic tsunami that in the subsequent hours reached the shores of many different countries causing terrible destructions. It killed some 23 0000 people, becoming one of the deadliest natural disasters in history.

The sheer magnitude of the phenomena defies imagination. The earthquake that triggered the tsunami involved 4×10^{22} joule. The tsunami itself carried 2×10^{18} joule,

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equivalent to a hypothetic 500 megaton nuclear bomb or to the consumption of 20 million homes in one year. And the energy per square meter discharged on some coastal areas exceeded that of the Hiroshima atomic bomb by two orders of magnitude.

Over the years, the analysis of the phenomenon revealed that many casualties and part of the destruction could have been prevented by a better knowledge of the underlying physics by the broad population and the political leaders. Just to give an example, consider this piece of news [9] released right after event: "*In Sri Lanka ... 80 per cent of coastal fishing vessels —nearly 20 000 boats— were completely destroyed or very seriously damaged*". This clashes with the fact (see later) that many boats could have been saved by simply rushing them offshore right after the tsunami alarm. Apparently, in 2004 only a few boats were saved with that countermeasure. During the 2011 tsunami in Japan, the strategy was applied by many fishermen based on their ancestral knowledge —and then became a recommended practice prepared in advance.

Acquiring a basic knowledge of the physics of a tsunami is, therefore, objectively important [1]. This applies in particular to high school and college students from all disciplines, for at least two reasons. First, they can act as knowledge disseminators in their communities. Second, with mass intercontinental tourism no one is totally safe, even if she/he lives in a country like Italy, almost immune from gigantic tsunamis. In fact, many victims in 2004 were unaware tourists from faraway places.

The desirability of broad dissemination clashes, however, with a reality: the water wave theories [10–14] normally used to treat tsunamis are complex and not suitable for students except those of master-level physics [1] —and certainly not for the general public. As a consequence, many presentations introduce the basic facts as assumptions, without justifying them. This can be acceptable in some cases, but a better knowledge of the underlying physics would be preferrable.

In 2005, we tackled this issue [1] by developing an original approach that derived the key properties of tsunamis in a simplified way. Over the past two decades, this work had a rather broad impact, being used by many websites, mass media and other entities as a standard elementary introduction to the subject. We believe that this dissemination also had a practical impact, notably influencing decisions about preventive countermeasures.

Here, we re-visit the issue with a revised approach, justified by two reasons. First, the continuing analysis of the 2004 tragedy put a new accent on facts that should be presented to students in a simple form. Second, the elementary formal derivation of the Tsunami properties also evolved, eliminating some idiosyncrasies of our 2005 article [1].

To make the material more useful for the target audience, we divided the presentation into two parts. The first one is a semi-qualitative discussion, particularly suitable for junior students and their teachers. The second presents instead recent formal developments, suitable for more advanced and/or curious students and bystanders.

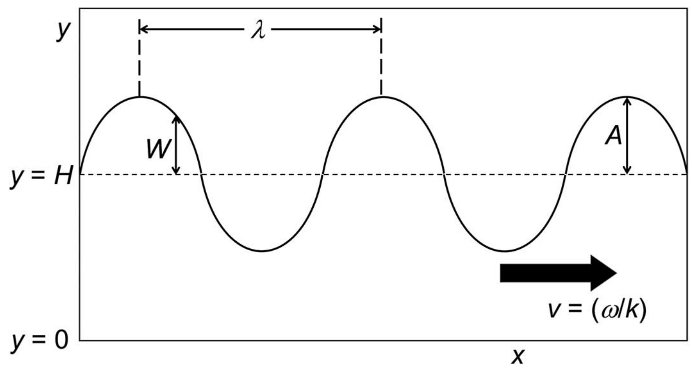

Fig. 1. – The simple sinusoidal water wave of equation (1).

2. Basic teaching instruments

A tsunami is a wave that propagates a perturbation of the water surface away from the site where it initially occurs. The possible triggering causes are different but all on very large scales: landslides, earthquakes, volcanic eruptions, possibly meteorites. In each case, the cause displaces an enormous water mass perturbing the previous equilibrium state. Then, this perturbation travels to faraway places carrying a very big amount of energy. If discharged on populated coastal areas, this energy causes massive damage and can kill many people.

One important fact is that a tsunami, being a wave, carries energy but it does not carry water. The affected water particles have local motions but, in first approximation, do not travel with the wave [1]. What travels, instead, is the perturbed shape of the upper water surface.

In our description of this motion, we shall work for simplicity in two dimensions as shown in fig. 1, using the coordinates x (horizontal) and y. In reality, however, water waves are three-dimensional phenomena propagating in two horizontal dimensions, like the circular waves caused by a stone falling in a lake. This notably decreases the energy per unit wavefront length as the distance from the initial site increases, limiting the impact on faraway places.

Assume that $y = 0$ at the water bottom and that H is the depth. A wave is described by the vertical distance W between its surface and the unperturbed water surface, $y = H$. W is a function of the time t and of x, and one simple mathematical form is

$$W = A\sin(kx - \omega t), \tag{1}$$

where k is the wavenumber and ω the angular frequency. One can easily realize that this waveform propagates with speed

$$v = \frac{\omega}{k}\,, \tag{2}$$

specifically, v is the "phase velocity", whereas the "group velocity" is defined as

$$v_g = \frac{\mathrm{d}\omega}{\mathrm{d}k}\,. \tag{3}$$

The wave period is $T = 2\pi/\omega$, and the space periodicity, *i.e.*, the wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi v}{\omega} = vT. \tag{4}$$

A real tsunami is of course much more complicated than the simple sinusoidal shape of equation (1) and fig. 1. Indeed, its initial cause produces a very complex water surface. But this shape can be mathematically analyzed, using the Fourier theorem, as a combination of sinusoidal waves like that of fig. 1.

3. The essential factors

The propagation of a tsunami is determined by a few factors, some of which must be included even in elementary descriptions, whereas others can be neglected. The first one is that the water density is constant (incompressibility). The second is that the water particles are subject to the force of gravity, which is conservative —*i.e.*, the sum of the potential and kinetic energies is constant in the corresponding water particle motions.

One factor that we can mostly neglect is turbulence, a phenomenon observed in everyday life. For example, rocks in a river break a steady flow of water producing foamy and bubbly currents that change with time. Turbulence causes losses of energy and is a very complicated phenomenon. But for a tsunami it is mostly negligible except when the wave reaches a shore [1–8, 10–14]. We shall also neglect viscosity — which is rather low for water— and other more complicated aspects like the possible "non-irrotational" character of the water particle velocities [10–14].

4. The key properties

Figure 2 schematically resumes the evolution of a tsunami from the creation to the arrival on a shore. A key aspect of this evolution is that the velocity in deep waters is simply given by

$$v = \sqrt{gH} \approx 3.1\sqrt{H}, \tag{5}$$

where $g \approx 9.8\,\mathrm{m/s^2}$ is the acceleration of gravity. We shall justify later this law in two different ways. Practically speaking, far from the coast the depth H is typically large and equation (5) corresponds to a very high speed. For example, $H = 5000\,\mathrm{m}$ gives $v = 221\,\mathrm{m/s} \approx 800\,\mathrm{km/hour}$ —comparable to a commercial passenger jet.

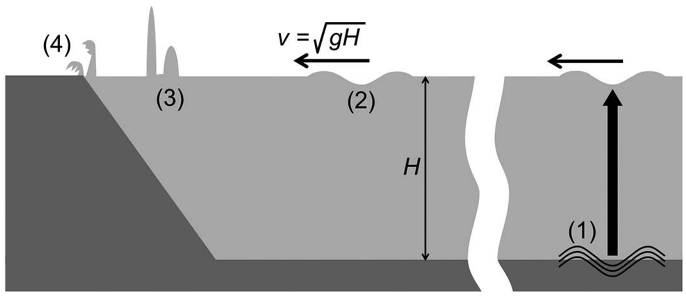

Fig. 2. – Schematic explanation of the four evolution phases of a tsunami: 1) creation, for example by an earthquake; 2) propagation over a long distance; 3) arrival at a shore: the amplitude increases and the wavelength decreases; 4) turbulence and inland penetration, limited because of the rapid energy losses.

A tsunami can thus travel very far in a relatively short time: it can cross the entire Pacific Ocean in less than one day. Luckily, however, the time to reach a distant shore is often long enough for launching alarms and activating effective countermeasures. Furthermore, if the initial cause produces seismic waves besides the tsunami, they travel faster and can give an earlier warning.

Close to a shore, H decreases and so does v according to equation (5). But the speed is still rather high: for $H = 10\,\mathrm{m}$, equation (5) would give $v \approx 10\,\mathrm{m/s}$, like the fastest Olympic sprinters. Thus, no one should try to outrun a tsunami, but get out of the shore area immediately after an alarm.

The second important characteristic of a tsunami is the period T. Since a real tsunami does not have a pure sinusoidal shape like fig. 1, we must define this parameter with a reasonable approach. In essence, the time scale corresponding to T is determined by the duration and properties of the phenomenon that creates the tsunami. For example, by the time length of a landslide or by the period, duration and area of earthquake vibrations.

In practice, the large size of the triggering phenomenon can bring the value of T to tens of minutes. This has an important consequence: the different phases of a tsunami can be far apart from each other, up to an hour or even more. Note that in many cases the first phase is "negative": the water leaves the beach. This unusual phenomenon can attract curious bystanders. But later a murderous, gigantic water peak arrives, striking them before they can run away.

The long period is linked to the third fundamental characteristic of tsunamis: very long wavelengths λ. According to equation (4), a period of tens of minutes and a speed of $800\,\mathrm{km/h}$ produce wavelengths of several hundred kilometers. These are much longer than those of "normal" water waves, most often caused by the wind —which range from less than a meter to a few hundred meters.

The enormous tsunami wavelengths have two very important physics consequences. First, they guarantee the validity of the velocity law of equation (5). In fact, this result only applies to "shallow-water" waves [1, 10–14], *i.e.*, when $\lambda \gg H$. For "normal" water waves this condition is not always valid, but for the large tsunamis wavelengths it works even in the deepest ocean areas.

Second, long wavelengths affect how fast tsunamis lose energy. This can be intuitively understood by comparing tsunamis —or normal waves— in deep waters and near the shore. In the second case, the wavelengths are reduced according to equations (4) and (5), and one sees that the waves become wildly turbulent, rapidly losing energy. Instead, in deep waters and with long wavelengths, the symptoms of turbulence are very limited and the energy losses are extremely low.

The theories of energy dissipation are quite complicated. But we can intuitively match the above qualitative observations by assuming —as it is commonly done— that the energy loss rate is inversely proportional to the wavelength [10–14]

$$\text{Energy loss rate} = \frac{\text{constant}}{\lambda}\,. \tag{6}$$

This law matches, in particular, the empirical observation that long wavelengths in deep waters correspond to very limited energy losses. However, we have seen that the energy is spread over a wavefront of increasing length, and this decreases the energy per meter. The wavefront length is approximately proportional to the distance from the source (think for example about circular waves), thus the energy per meter is inversely proportional to the same distance: this is the fourth fundamental characteristic of a tsunami.

The fifth one concerns the amplitude of the tsunami wave, A. As we shall demonstrate below, A changes with the water depth according to

$$A = \text{constant} \times \sqrt[4]{\frac{1}{H}}\,. \tag{7}$$

This law has two fundamental consequences: first, it makes it difficult to detect a tsunami in deep waters, where the amplitude corresponds to a rather gentle wave. Second, it makes its arrival on a shore potentially devastating, since as H decreases A increases. Consider, for example, a killer tsunami with $A \approx 10$ meters at a shore, say for $H = 2\,\text{m}$. When it was in deep waters, *e.g.*, for $H = 5000\,\text{m}$, its amplitude was $A \approx 1.4\,\text{m}$, hardly noticeable and practically indistinguishable from a "normal" wave.

Finally, note that the combination of equations (4), (5) and (6) boosts the energy losses when a tsunami reaches a shore. This does not allow its energy and its water to penetrate inland much beyond the shoreline: the inundation is typically limited to a few hundred meters. This is a very positive fact for effective countermeasures. However, the short penetration also means that a lot of energy is discharged over a limited area, causing extreme devastation.

5. Derivation of key properties

As mentioned, full theories of tsunamis as water waves [10–14] are very complicated and much beyond the level of non-physics students or of the general public. Even with simplifying assumptions, for example those of the "Airy waves" linear model, understanding them requires a sophisticated background in mathematics and physics, including advanced concepts like the "velocity potential".

Our article of 2005 [1] presented alternate and rather elementary derivations of the key laws of tsunami physics, equations (5), (6) and (7). We later realized, however, that the level was still too complicated for a non-specialized audience. For example, the derivation of equation (5) required advanced notions of water dynamics and used a formal device based on stationary waves [15], quite clever but not corresponding to real situations and therefore beyond intuition.

Some of these idiosyncrasies are eliminated by the revised derivation of the speed law (see below). But the results are not fully satisfactory from a teaching point of view since, for example, the role of water incompressibility is not explicitly visible. We thus also propose a second derivation of the speed law, directly based on incompressibility and energy conservation.

6. Motion of water particles

As shown in fig. 3, a wave causes the water particles to circulate along elliptical trajectories [1, 10–14]. At the upper surface, the vertical amplitudes of these motions coincide with that of the wave, A. Below, they decrease as the depth increases. And for shallow-water waves, they become zero at the bottom.

One key fact is that for gravity-caused motions of this kind (like for a pendulum) the average kinetic equals the average potential energy —if this latter is measured with respect to equilibrium. The property also applies to the entire tsunami: its total kinetic energy K equals the total potential energy U measured from equilibrium.

7. Deriving the amplitude-depth law

We shall now consider a simple "model" tsunami with the waveform shown in fig. 4a —which is a one-wavelength portion of the wave of fig. 1. The potential energy corresponds to raising the water from the "trough" to the "crest". The mass of water that is raised is proportional to the volume, in turn proportional to the wave amplitude A. This mass is raised vertically by a distance also proportional to A. Thus, the total potential energy is proportional to gA^2. And, since $U = K$, the total tsunami energy is also proportional to gA^2.

We can use this result to justify equation (7). Before energy losses become important at a shore, we can assume in first approximation that the energy flux of a tsunami is constant. The flux is the total energy multiplied by the velocity of equation (5). Therefore, it is proportional to $gA^2\sqrt{gH}$.

A constant energy flux thus requires $A^2\sqrt{H}$ and therefore A^4H not to change, justifying equation (7). In essence, it is energy conservation that forces the wave

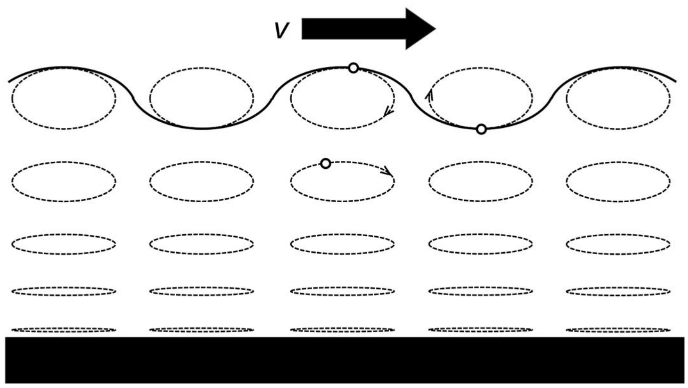

Fig. 3. – The particles of water affected by a tsunami wave, from the surface to the bottom, move along local and flat elliptical paths, whose vertical amplitudes decrease with the distance from the upper surface and become zero at the bottom. Note that the elliptical motions are local: the particles do not follow the wave —which propagates a perturbation and carries energy, but not water.

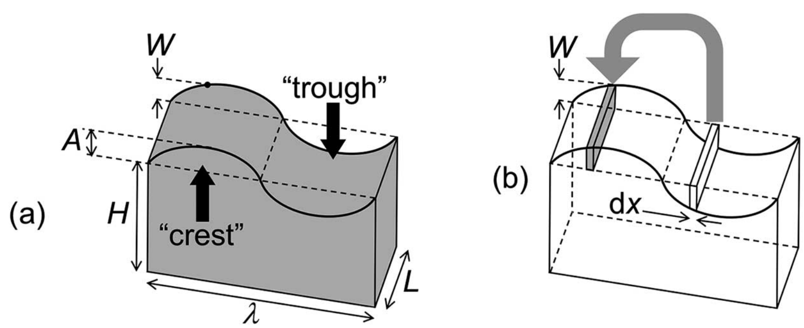

Fig. 4. – Analysis of a one-wavelength portion of the sinusoidal tsunami wave of equation (1).

amplitude to increase at the shore, transforming a rather gentle wave into a mass murderer.

8. Deriving the speed law

Imagine in the "model" tsunami of fig. 4 a slab of (infinitesimal) thickness dx and section of area LW (fig. 4b). The corresponding water mass is the density times the volume, $\rho LW\,\mathrm{d}x$. When raised from the trough to the crest, its potential energy increases by

$$(\rho LW\,\mathrm{d}x)gW = (\rho LW^2\,\mathrm{d}x)g = \rho gLA^2\sin^2(kx)\mathrm{d}x.$$

The total potential energy is the integral of this quantity over the raised volume length $\lambda/2$

$$U = \int_0^{\lambda/2}\rho gLA^2\sin^2(kx)\mathrm{d}x = \frac{\rho gLA^2}{k}\int_0^{\pi}\sin^2(kx)\mathrm{d}(kx) = \frac{\rho gLA^2}{k}\frac{\pi}{2} = \frac{\rho gLA^2\lambda}{4}. \tag{8}$$

This, by the way, formalizes the above conclusion that U is proportional to A^2.

We must now calculate the total kinetic energy for the water particle movements in the entire region from the surface to the bottom. For the elliptical motion of a given particle, the coordinates X and Y are

$$X = X_0(y)\cos(-\omega t), \qquad Y = Y_0(y)\sin(-\omega t). \tag{9}$$

Note [1] that: 1) at the top, $Y_0(H) = A$; 2) at the bottom, $Y_0(0) =$ zero; 3) $Y_0 \ll X_0$, producing flat ellipses. Following [1] and the empirical evidence, we can satisfy conditions (2) and (3) by assuming that

$$Y_0(y) \approx X_0(y) ky. \tag{10}$$

Therefore, the velocity components (using equation (2)) are

$$V_x = X_0\omega\sin(-\omega t) = \frac{Y_0}{ky}\omega\sin(-\omega t) = \frac{Y_0 v}{y}\sin(-\omega t), \tag{11}$$

$$V_y = -Y_0\omega\cos(-\omega t). \tag{12}$$

The shallow-water condition $\lambda \gg H$ implies that $k = 2\pi/\lambda$ is small and $V_x^2 \gg V_y^2$, so in the kinetic energy we can neglect V_y^2 and, for a particle of mass m

$$\text{particle kinetic energy} \approx \frac{m}{2}V_x^2 = \frac{m}{2}\left(\frac{Y_0}{y}\right)^2 v^2\sin^2(-\omega t), \tag{13}$$

whose average can be calculated using the averages of Y_0, y and of $\sin^2(-\omega t)$ (over time), which are $A/2$, $H/2$ and $1/2$

$$\text{average kinetic energy of a particle} = \frac{m}{4}\left(\frac{A}{H}\right)^2 v^2. \tag{14}$$

The total kinetic energy is obtained by replacing m in equation (14) with the total mass of the moving particles, *i.e.*, the mass $\rho LH\lambda$ in the volume $LH\lambda$. Therefore

$$\text{total kinetic energy} = \frac{\rho LH\lambda}{4}\left(\frac{A}{H}\right)^2 v^2 = \frac{\rho LA^2\lambda}{4H}v^2. \tag{15}$$

Since the total kinetic and potential energies of a tsunami are equal, equations (8) and (15) imply

$$\frac{\rho g LA^2\lambda}{4} = \frac{\rho LA^2\lambda}{4H}v^2, \tag{16}$$

and $v^2 = gH$, demonstrating equation (5).

Note, by the way, that the same result is also valid for the group velocity

(17) $$v_g = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \sqrt{gH}.$$

9. The speed law from incompressibility

As mentioned, we propose here a second original derivation in which the role of incompressibility besides energy conservation is clearly visible. The procedure is inspired by the wave theory of [14], with some relevant conceptual differences and adopting the shallow-water limit during the derivation rather than at its end. We assume that the two velocity components for a particle at the horizontal position x that shifts in time with the wave are ([1])

(18) $$V_x = V_{x0}(y)\sin(kx - \omega t),$$

(19) $$V_y = V_{y0}(y)\cos(kx - \omega t).$$

Consider now water incompressibility: imagine (fig. 5) an infinitesimal volume with sizes $\mathrm{d}x$, $\mathrm{d}y$ and $\mathrm{d}z$, and assume that the velocity along z is zero. During a time $\mathrm{d}t$, the water mass entering this volume along x (fig. 5a) —*i.e.*, across the surface $\mathrm{d}y\,\mathrm{d}z$— is $\rho \mathrm{d}y\,\mathrm{d}z V_x(x)\mathrm{d}t$, where ρ is the water density. The mass leaving the volume along x (fig. 5b) is

$$\rho \mathrm{d}y\,\mathrm{d}z V_x(x + \mathrm{d}x)\mathrm{d}t = \rho\,\mathrm{d}y\,\mathrm{d}z\left[V_x(x) + \frac{\mathrm{d}V_x}{\mathrm{d}x}\,\mathrm{d}x\right]\mathrm{d}t,$$

and the net flux along x is the difference between the masses entering and leaving the volume:

$$-\rho\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\left(\frac{\mathrm{d}V_x}{\mathrm{d}x}\right)\mathrm{d}t.$$

A similar conclusion is valid along the y-direction (figs. 5c and 5d), so the total net flux is

$$-\rho\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\left(\frac{\mathrm{d}V_x}{\mathrm{d}x} + \frac{\mathrm{d}V_y}{\mathrm{d}y}\right)\mathrm{d}t.$$

Due to incompressibility, the mass in $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$ cannot change and the net flux must be zero, thus

(20) $$\frac{\mathrm{d}V_x}{\mathrm{d}x} + \frac{\mathrm{d}V_y}{\mathrm{d}y} = 0.$$

This is the fundamental "continuity equation".

([1]) Note that these equations are not in conflict with equations (7) and (8) of [1], which apply to the *same* particle and do not depend on the horizontal position. Equations (18) and (19) apply instead to a horizontal position traveling with the wave, and therefore to continuously changing particles. Also note that, in general, the phase shift between the functions of equations (18) and (19) could be different from $\pi/2$ (the phase between a sine and a cosine, whose adoption simplifies the mathematical steps). However, the use of a generic phase shift would not change the conclusion derived here, *i.e.*, equation (5).

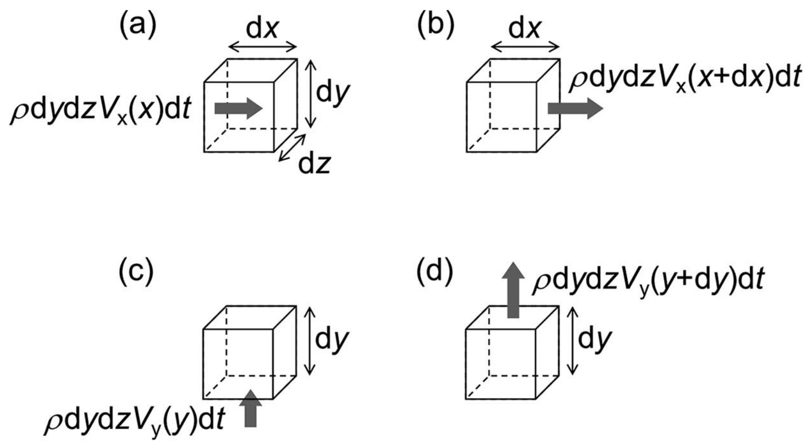

Fig. 5. – Derivation of the "continuity" equation (20) caused by the constant water density.

With the velocity components of equations (18) and (19), equation (20) gives

$$kV_{x0} = -\frac{\mathrm{d}V_{y0}}{\mathrm{d}y}\,. \tag{21}$$

Consider now that without the wave the particles would not move, their weights being offset by Archimedes' buoyant force. Therefore, equations (18) and (19) imply additional forces with components f_x and f_y that cause accelerations. Since the energy is conserved, such forces correspond to a potential energy u, so that

$$-\frac{\mathrm{d}u}{\mathrm{d}x} = f_x = m\frac{\mathrm{d}V_x}{\mathrm{d}t} = mV_{x0}[-\omega\cos(kx-\omega t)], \tag{22}$$

$$-\frac{\mathrm{d}u}{\mathrm{d}y} = f_y = m\frac{\mathrm{d}V_y}{\mathrm{d}t} = mV_{y0}\omega\sin(kx-\omega t). \tag{23}$$

Equations (22) and (23) can be solved with a function u similar to equation (1)

$$u = u_0(y)\sin(kx-\omega t), \tag{24}$$

so that equation (22) gives

$$\begin{aligned} &-u_0k\cos(kx-\omega t) = mV_{x0}(-\omega)\cos(kx-\omega t),\\ &u_0k = mV_{x0}\omega, \end{aligned} \tag{25}$$

and from equation (23) we get

$$\frac{\mathrm{d}u_0}{\mathrm{d}y} = -mV_{y0}\omega. \tag{26}$$

From this last equation, we can derive the function $u_0(y)$; in fact

$$(27)\qquad \begin{aligned}\frac{\mathrm{d}^2u_0}{\mathrm{d}y^2} &= -m\frac{\mathrm{d}V_{y0}}{\mathrm{d}y}\omega = [\text{considering equation (21)}] = mkV_{x0}\omega\\ &= [\text{considering equation (25)}] = k^2u_0,\\ \frac{\mathrm{d}^2u_0}{\mathrm{d}y^2} &= k^2u_0.\end{aligned}$$

We can propose as solutions of equation (27) either $u_0 = C\exp(ky)$ or $u_0 = C\exp(-ky)$, where C is a constant. But we must also satisfy the condition of zero vertical velocity at the sea bottom. Thus, equation (26) requires the y-derivative of u_0 to be zero for $y = 0$ —a condition satisfied by combining the two above solutions for u_0:

$$(28)\qquad u_0 = C[\exp(ky) + \exp(-ky)].$$

Consider now two other boundary conditions. First, at the top ($y = H$) the vertical velocity must match that given by the wavefunction of equation (1)

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -A\omega\cos(kx-\omega t) = V_y(H) = V_{y0}(H)\cos(kx-\omega t),$$

which, considering equations (26) and (28), gives

$$(29)\qquad \begin{aligned}&-A\omega = V_{y0}(H) = -\frac{1}{m\omega}\left(\frac{\mathrm{d}u_0}{\mathrm{d}y}\right)_{y=H} = -\frac{1}{m\omega}Ck\big[\exp(kH) - \exp(-kH)\big],\\ &\frac{m\omega^2}{k}A = C\big[\exp(kH) - \exp(-kH)\big].\end{aligned}$$

The second boundary condition is that at the water top the potential energy corresponds to the vertical displacements produced by the wave, *i.e.*, $u_0(H) = mgA$, or

$$(30)\qquad C[\exp(kH) + \exp(-kH)] = mgA.$$

In the case tsunamis, the wavelength is very long, so $kH = 2\pi H/\lambda$ is very small and $\exp(\pm kH) \approx 1 \pm kH$, so equations (29) and (30) approximately become

$$(31)\qquad C(2kH) \approx \frac{m\omega^2}{k}A,$$

$$(32)\qquad 2C \approx mgA.$$

Combined together, these equations give:

$$v = \frac{\omega}{k} \approx \sqrt{gH}\,,$$

i.e., the speed law of equation (5).

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