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A historically correct didactic first step in the quantum world: stressing the interplay of relativity, thermodynamics and quantum physics

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Abstract
Quantum physics is the backbone of modern science: therefore, a correct first step is essential for students' success in many different disciplines. Unfortunately, many didactic approaches are still complicated, potentially confusing and often historically wrong. An alternate, simple, stimulating and historically correct approach is outlined here.

1. Standard didactic difficulties in stepping into the quantum world

Quantum physics, relativity and thermodynamics are the pillars of all branches of modern science, therefore their didactic introduction is of critical importance in all scientific curricula. Quite interestingly, they are almost always presented as independent topics. The point of this paper is to show that excellent didactic opportunities are missed in this way, based on the specific example of Einstein's photon hypothesis [1, 2].

Relativity, quantum physics and thermodynamics do have very strong links that manifest themselves even at the most elementary levels [1]. Molecular specific heats cannot be understood without the quantum freezing of some degrees of freedom. The relativistic Doppler effect is arguably the historical gate that led Einstein to the quantum concept of the photon [1, 2]. In turn, photons are required to understand why the entropy of electromagnetic radiation is similar to that of an ideal gas [2], and so on....

The artificial separation of such strongly linked topics might not be an excessive price to pay if one could thus achieve an effective didactic introduction to quantum concepts. I believe that the contrary is true: most standard introductions to quantum physics are difficult to follow (in particular for non-physics students), historically incorrect and—frankly—quite boring.

Consider, for example, a frequently used strategy: to begin the discussion of quantum concepts with the energy quantization required to explain the blackbody spectrum. No elementary-level student, in my opinion, can follow the corresponding derivation without finding it rather difficult. Furthermore, Planck's theory was not really the first step into quantum physics but a last-ditch attempt to salvage classical physics with a very clever trick.

My teaching experience indicates that the best way to begin quantum physics is to discuss the first real quantum hypothesis, Einstein's photon [2]. This strategy, however, must avoid a common mistake: Einstein's supposed derivation of the photon hypothesis from data on the photoelectric effect, which is a historical absurdity. Such data, in fact, did not exist at the time of Einstein's work in 1904–1905. The works of Hertz in 1887, Thompson in 1899 and Lenard in 1892 had discovered the photoelectric effect, related it to electrons and demonstrated that the electron energy is independent of the light intensity [3]. However, suitable photoemission experiments to test Einstein's photon hypothesis took place much later, in 1915 [4].

This historical distortion is not only bad *per se* but also a wrong didactic strategy because it conceals the truly revolutionary character of Einstein's photon hypothesis [1]. The alternate didactic approach presented here is based on a simplified version of Einstein's true path to the photon hypothesis. Practical tests in my own classes show that the approach is effective and stimulating besides being historically correct.

2. Elements for an alternate approach

The approach requires the students to already possess understanding of some elementary notions in relativity, thermodynamics and electromagnetism, and specifically:

(1) the existence of electromagnetic waves of different frequencies and the relation between wavelength and frequency;
(2) the fact that electromagnetic waves carry energy E and momentum $\boldsymbol{p}$ of magnitude p, with $p = E/c$;
(3) the Lorentz transformations and their most important consequences;
(4) the Wien displacement law for blackbody radiation;
(5) in general, the possibility of applying thermodynamic approaches to the electromagnetic radiation, established in the physics community by Max Planck.

Some of these notions may not have been previously presented to the students; in that case, the didactic strategy outlined here should be preceded by a careful treatment of them.

One important element of the didactic strategy is a description of the historical context in 1905, stressing the following points.

- Notions that are universally accepted today were still controversial in 1905. These included the use of statistical mechanics of atoms and molecules to explain thermodynamics and, in fact, the very existence of atoms.
- Classical physics was still very much in the saddle. The problems arising from the blackbody spectrum were not considered as the breaking point of classical physics, but just a temporary problem to be solved within its framework. Therefore, Planck's blackbody theory was not considered a departure from classical physics but something to be reabsorbed into it; Planck himself tended to adopt this opinion.
- Experiments on electrons (that had been discovered only a few years before) were still quite primitive. In particular, the few available results on the photoelectric effect did not yet include the frequency threshold for photoemission.
- Einstein's works on the photon, on relativity and on Brownian motion [2] were all published in 1905 within a few months of each other—and their conceptual development took place in parallel with significant interactions between the three works [1].

As a human touch, one could also mention that young Albert Einstein had finished his PhD work but had not yet obtained an academic appointment. His job as a Swiss public-service employee was quite secure but did not include research (or teaching) duties. Nevertheless, he was very active in research, but as an outsider with respect to the academic world he could not afford to make serious mistakes without jeopardizing his credibility.

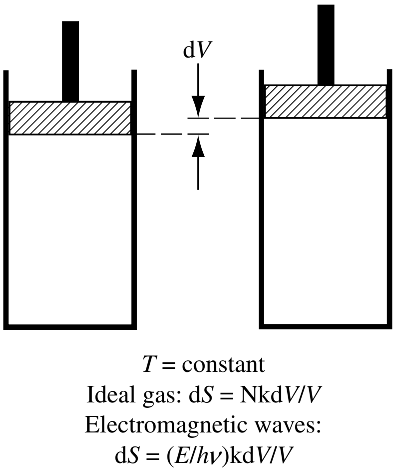

Figure 1. The similarity between the entropy changes for an ideal gas and for electromagnetic waves led Einstein to hypothesize the existence of photons.

3. Didactic strategy

The logic line is a very simplified version of Einstein's own path to the photon hypothesis [2]. The first step is the standard calculation of the entropy change for an ideal gas at constant temperature. The process analysed is the expansion of N particles of ideal gas from a volume V_0 to a volume V, formed by two containers of volume V_0 and $(V - V_0)$ separated by a valve.

After the valve is opened, the probability for one gas particle to be found in the original recipient V_0 is V_0/V. For N particles, the probability of all being found in V_0 is $(V_0/V)^N$. The corresponding Boltzmann entropy is $k\ln(V_0/V)^N = Nk\ln(V_0/V)$.

Since the probability of the final state is very close to unity and the corresponding entropy near zero, the entropy change is $\Delta S = -Nk\ln(V_0/V) = Nk\ln(V/V_0)$. After differentiating with respect to V this becomes

$$\mathrm{d}S = Nk(\mathrm{d}V/V). \qquad (1)$$

This is the well known entropy change of an ideal gas for an infinitesimal change of volume (figure 1).

Imagine now (figure 1 again) a container filled with electromagnetic waves. Specifically, the waves first fill the volume V_0 and are then allowed to expand at constant temperature. What is the entropy change?

Consider a hypothetical reversible process that expands the volume by $\mathrm{d}V$. The entropy change is $\mathrm{d}S = \delta Q/T = \mathrm{d}E/T + P\,\mathrm{d}V/T$. The experiments on blackbody radiation show that the internal energy only depends on the temperature: $E = E(T)$, so that for constant temperature $\mathrm{d}E = 0$ and $\mathrm{d}S = P\,\mathrm{d}V/T$.

The pressure P created by the electromagnetic waves can be easily calculated by taking a unit area of the container surface. The perpendicular momentum carried by the incoming waves in a time $\mathrm{d}t$ is $\mathrm{d}p = [(E/V)/2c]c\,\mathrm{d}t$—where the factor of two takes into account the two possible directions of the wave (towards the surface and away from it). The momentum change per unit area is then $2\mathrm{d}p = (E/V)\,\mathrm{d}t$ and the pressure is $P = (E/V)$.

Taking this into account, the entropy change is $\mathrm{d}S = (E/T)\,\mathrm{d}V/V$. On the other hand, the Wien displacement law relates the wavelength (and therefore the frequency) of the waves and their temperature. The wavelength of maximum energy density is $\lambda = \text{constant}/T$, and the corresponding frequency is $\nu = \alpha T$, where α is a constant.

We can now adopt a very rough but effective approximation in which the frequency distribution of the waves in the cavity is replaced by one single frequency corresponding to the maximum energy density[1]. The entropy change becomes

$$\mathrm{d}S = (\alpha E/\nu)(\mathrm{d}V/V). \tag{2}$$

In order to compare equations (1) and (2), it is convenient to write the constant α as $\alpha = k/h$, where h is a (yet undefined) constant; equation (2) becomes

$$\mathrm{d}S = [E/(h\nu)]k(\mathrm{d}V/V). \tag{3}$$

The comparison of equations (1) and (3) (figure 1) led Einstein to the photon hypothesis: the entropy change for electromagnetic waves is the same as for an ideal gas, as long as we assume that $E/(h\nu)$ is the number of 'gas' particles—and therefore that $h\nu$ is the energy of a light (electromagnetic wave) particle.

Specifically, Einstein's revolutionary hypothesis was to consider this result not just a mathematical artifact, but the manifestation of the real quantum nature of light. He thus used this hypothesis to predict (and not to justify) the properties of the photoelectric effect.

Einstein's courage in making this step defies imagination. Consider, in fact, that:

- The (relatively) recent success of Maxwell's theory, corroborated by Hertz's experiments, had seemingly put a final seal to the question of the nature of light in favour of a wave nature. Einstein could not dismiss this evidence, but boldly accepted the apparent contradiction as a fact.
- If wrong, the photon hypothesis would have destroyed any future chances of academic career for its young author. The 'safe' way would have been to publish the much less controversial works on relativity and on Brownian motion [2], and let the photon hypothesis rest for a while. Fortunately for us, Einstein had the incredible courage to go the other way.

3.1. How could Einstein be so bold?

Where did Einstein find this courage? The answer, in my opinion, must be found in an amazing comment he made in the relativity article [2] of 1905: '*Es ist bemerkenswert, daßdie Energie und die Frequenz eines Lichtcomplexes sich nach demeselben Gesetze mit dem Bewegungszumstande des Beobachters ändern*' ('*It is remarkable that the energy and the frequency of a light complex vary with the state of motion of the observer in accordance with the same law*').

In order to understand this comment, consider two different inertial reference frames with relative speed v. The elementary treatment of the (relativistic) Doppler effect shows that the frequency transforms as

$$\nu' = [(1-\beta)/(1+\beta)]^{1/2}\nu; \tag{4}$$

on the other hand, the general Lorentz transformation law for energy is $E' = \gamma(E - c\beta p)$. For an electromagnetic wave $p = E/c$, thus $E' = \gamma E(1-\beta)$, which gives

$$E' = [(1-\beta)/(1+\beta)]^{1/2}E. \tag{5}$$

Equations (4) and (5) show that the frequency and the energy of an electromagnetic wave are Lorentz-transformed in the same way. This point was laconically but revealingly stressed by Einstein in the above-quoted comment [1, 2]—and, in my opinion, gave him the necessary boldness to propose photons not as theoretical artifacts but as real things.

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[1] This is a crude approximation indeed, whose only justification is to keep the presentation at a reasonably elementary level and to avoid distracting the students with excessive formalism. The students should be told, however, that Einstein's actual derivation was not based on this hypothesis but on the correct expression of the entropy of radiation as a function of frequency and of the energy density, which follows from Wien's law. Students interested in the rigorous treatment should be directed to Einstein's original paper of 1905 and/or to historical texts.

3.2. The magnitude of quantization

This is an essential part of the teaching strategy: the result that $h\nu$ is the photon energy must be immediately followed by simple arguments to place this energy on a concrete scale. The best approach is to use facts from common experience.

Most students are aware, for example, that microwaves, infrared light, visible light, ultraviolet light and x-rays have different effects. After deriving the corresponding frequencies from the wavelengths, the teacher can discuss effects such as ‘cooking’, ‘warming up’, ‘tanning’ and ‘ionizing’ in order to class them in two broad categories; those which do not alter chemical bonds and core electrons and those which do. The boundary between the two categories can thus be roughly placed between the visible and the ultraviolet, at frequencies of the order of 10^{15} s^{-1}.

The typical energy of a chemical bond can then be roughly estimated from the order of magnitude of the melting latent heat of a solid, by dividing it by the number of chemical bonds per unit weight. The resulting estimate is of the order of 10^{-18} J per bond, which combined with the above frequency of 10^{15} s^{-1} leads to a rough estimate $h \approx 10^{-33}$ J s.

This magnitude can then be used to explain why the quantization of light cannot be directly seen in common experience. For example, the energy of a visible photon can be compared to other typical energies such as the specific heat of our body or the energy delivered by a lamp within 1 s.

4. Concrete remarks and conclusions

The above teaching approach offers several advantages with respect to other standard methods:

- it is very simple and free of complicated mathematical formalism;
- it closely follows Einstein’s true path towards the notion of photons [2];
- it puts the birth of quantum physics in a correct historical perspective by emphasizing Einstein’s photon as the first real quantum hypothesis;
- it is based on simple notions about gases and waves that should be familiar to the students;
- it immediately leads to quantitative estimates.

Last but not least, it stimulates the students to imagine a young Albert Einstein—close to their age—struggling both with revolutionary ideas and with his own career. This makes Einstein much more human and interesting than the iconized saint-like old-man image cherished by popular literature.

Acknowledgments

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