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Quantization, Doppler shift and invariance of the speed of light: some didactic problems and opportunities

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Abstract. I discuss some of the didactic problems in the introduction to quantum physics based on the concept of photon. I specifically argue that the derivation of the photon energy from the relativistic Doppler shift offers stimulating didactic opportunities. A detailed form of this derivation reveals the subtle link between relativity and quantization, and also helps in elucidating the meaning of the invariance of the speed of light.

Résumé. J'analyse quelques-uns des problèmes didactiques liés à l'introduction à la physique quantique se basant sur la notion de photon. Je présente spécifiquement des arguments qui montrent que la dérivation de l'énergie du photon à partir de l'effet Doppler relativiste crée des intéressantes ouvertures didactiques. Une forme détaillée de la dérivation révèle les liaisons subtiles entre relativité et physique quantique, et peut nous aider à clarifier l'interprétation de l'invariance de la vitesse de la lumière.

1. Preliminary remarks

My teaching experience in modern physics courses suggests that the most effective introduction to quantum physics is not the 'historical' approach based on the blackbody theory, which is quite difficult for students to grasp. I much prefer to begin with an empirical introduction to the concept of photon, which I then use to present the dual nature of photons, leading to the dual nature of other particles, to the wave properties of electrons, to the Bohr atom, etc.

The concept of photon can be introduced empirically, for example by showing a single-photon-counting detection system (or a video presentation of the same). In the past, I subsequently derived the photon energy, $Q = h\nu$, from the photoelectric effect. This approach leads to some practical and conceptual difficulties; furthermore, it tends to perpetuate the misconception that Einstein derived the $Q = h\nu$ relation from the photoelectric effect. This would have been impossible with the data quality of 1905, and in fact Einstein's derivation was purely thermodynamic. For a discussion of this point see Margaritondo (1988).

There is an alternate approach for deriving the photon energy, which I would like to propose to colleague teachers because I find that it offers stimulating didactic opportunities. Specifically, it can be used to elucidate the subtle links between relativity and quantum physics, and also to clarify the principle of the invariance of the speed of light.

In its most straightforward form, the argument is the following. The general Lorentz transformation of the energy is $W' = \gamma(W - \beta cp)$, where p is the momentum corresponding to the energy W. On the other hand, the relation between energy and momentum for an electromagnetic wave is given by the classical-physics theory of electromagnetism: $p = W/c$.

Therefore, for an electromagnetic wave the Lorentz transformation of the energy becomes $W' = W\gamma(1-\beta) = \sqrt{(1-\beta)/(1+\beta)}$. This is the same transformation law valid for the wave's frequency, i.e. the relativistic Doppler shift: for an electromagnetic wave both energy and frequency Lorentz-transform in the same way.

This implies that if we now suppose that the wave's energy is quantized, the quantum Q will also Lorentz-transform like the frequency. Thus, the

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relation between Q and the frequency must be linear and can be written as $Q = h\nu$.

This approach to the argument, however, is a bit too formal and it misses in part the aforementioned didactic opportunities. I prefer a longer but more stimulating approach, based on a detailed analysis of a simple experimental situation.

2. The empirical approach

Consider a monochromatic plane wave propagating along the x-axis in an inertial reference frame F, which is detected by a photomultiplier. Assume that the active surface of the photomultiplier is of area Σ and perpendicular to the x-axis. Also assume that the wave is linearly polarized, with time-averaged electric field magnitude E. Its energy density (in MKS units) is given by:

$$\rho = \tfrac{1}{2}\epsilon_0(E^2 + c^2B^2) = \epsilon_0 E^2. \qquad (1)$$

The wave's energy reaching the photomultiplier during a given time interval Δt is:

$$\Omega = c\rho\Sigma\Delta t. \qquad (2)$$

Assume now that the wave's field is quantized, in the sense that the detected energy by the photomultiplier is always a multiple of the photon energy, Q. This means that the photomultiplier 'clicks' N times during Δt, with:

$$N = \frac{\Omega}{Q} = \frac{c\rho\Sigma\Delta t}{Q}. \qquad (3)$$

Consider now how the same phenomenon would appear in a second inertial reference frame F', moving along the positive direction of the x-axis with speed u. The number of times N' the photomultiplier 'clicks' when seen from F' can be derived by assuming that the photon energy in F' is Q', and noting that the time interval Δt becomes $\Delta t' = \gamma\Delta t$, whereas the area Σ is invariant.

The wave's energy density ρ' in F' can be obtained from equation (1), by using the Lorentz transformations for the wave's (transverse) E and B-fields: $E' = \gamma(E - uB) = \gamma(1-\beta)E$, therefore:

$$\rho' = \epsilon_0 E'^2 = \epsilon_0\gamma^2(1-\beta)^2E^2 = \gamma^2(1-\beta)^2\rho. \qquad (4)$$

In order to calculate N', one must take into account that the photomultiplier moves in F' with speed u along the negative direction of the x'-axis—as schematically shown in figure 1. This brings us to an apparent paradox that we further discuss later: seen from F', the difference of the wave's speed and of the detector's speed is *not* c, but $(c+u)$. Thus, in F', the wave's energy reaching the photomultiplier during $\Delta t'$ is not $c\rho'\Sigma\Delta t'$, but:

$$\Omega' = (c+u)\rho'\Sigma\Delta t' = c(1+\beta)\rho'\Sigma\Delta t', \qquad (5)$$

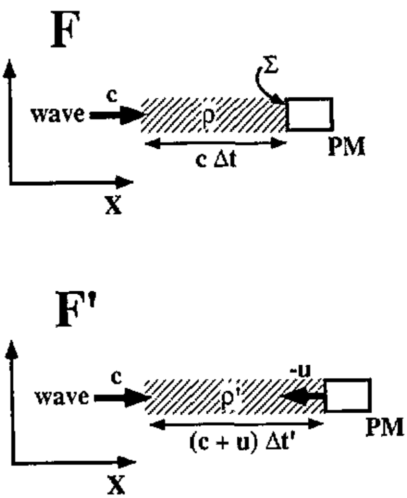

Figure 1. Schematic diagram for the calculation of the number of times the photomultiplier PM 'clicks' when seen from the two reference frames: in the frame F, during the time period Δt, the wave's energy reaching PM is $\rho c\Delta t\Sigma$, where ρ is the wave's energy density in F and $c\Delta t\Sigma$ is the wave's 'volume' (shaded area) reaching PM during Δt. This gives equation (2). In F', because of the motion of PM with velocity $-u$, the equivalent expression is $(c+u)\Delta t'\rho'\Sigma$.

and

$$N' = \frac{\Omega'}{Q'} = \frac{c(1+\beta)\rho'\Sigma\Delta t'}{Q'} = \frac{c(1+\beta)\gamma^2(1-\beta)^2\rho\Sigma\gamma\Delta t}{Q'} = \frac{c\rho\Sigma\Delta t}{Q'\sqrt{\dfrac{1+\beta}{1-\beta}}}. \qquad (6)$$

On the other hand, the number of times the photomultiplier 'clicks' must be invariant from one reference frame to the other, thus $N = N'$, and equations (3) and (6) give:

$$Q' = Q\sqrt{\frac{1-\beta}{1+\beta}}. \qquad (7)$$

This relation shows that the Lorentz transformation for the photon energy is the same as that of the wave's frequency, i.e. the relativistic Doppler shift—and leads to $Q = h\nu$ and $Q' = h\nu'$.

3. The didactic opportunities

The above version of the argument, although somewhat lengthy, is in my opinion preferable because it opens up interesting didactic opportunities. First of all, it uses in each step a realistic experiment, thereby bringing a sense of reality into the formal relation between the relativistic Doppler effect and the photon energy, i.e. between relativity and quantum physics.

Furthermore, a key point in the derivation is the correct interpretation of the invariance of the speed of light, which removes the apparent paradox mentioned in the previous section. One can emphasize this point by asking the students' opinion on a statement like: 'Consider a light beam and a detector; observed from the detector's rest frame, the light's speed is c. Take now another inertial frame in motion with respect to the first. Can one say that in this frame the *difference* of the velocity of light and of the detector's velocity is still c?'

The answer is obviously no, and one can convince the students in three ways. First, because a difference equal to c would lead the argument of the previous section to an incorrect conclusion. Second, one can actually derive the Lorentz transformation of the difference of two velocities, $(\mathrm{d}x_1/\mathrm{d}t) - (\mathrm{d}x_2/\mathrm{d}t)$, then assume $(\mathrm{d}x_1/\mathrm{d}t) = c$ and $(\mathrm{d}x_2/\mathrm{d}t) = 0$, and show that the Lorentz-transformed difference in our case is indeed $c + u$.

Third, one can (and should) discuss in detail the meaning of the relativistic invariance of the speed of light, explaining why it does not directly concern this specific case. Specifically, one should point out that the invariance concerns the speed of the wave measured in F' and the speed of the wave measured in F—and show how it can be easily derived from the relativistic combination of velocities. Then, one should point out that the *difference*—measured in F'—between the wave's and multiplier's velocities is *not* the speed of the wave measured in F'—and therefore is not required to be invariant and equal to c. In conclusion, whereas the combination of the light's and reference frame's speeds always gives c, the same is not true for the combination of the light's and another object's speeds.

In my opinion, these didactic opportunities make the argument quite stimulating for the students. I would like to conclude with a personal experience: at one time, I proposed an open competition (with a prize) on the apparent paradox, to which even senior colleagues were happy to participate.

Acknowledgments

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Reference

Margaritondo G 1988 *Physics Today* **41**(4) 66–70 and the references therein